Special Delivery: Programming with Mailbox Types
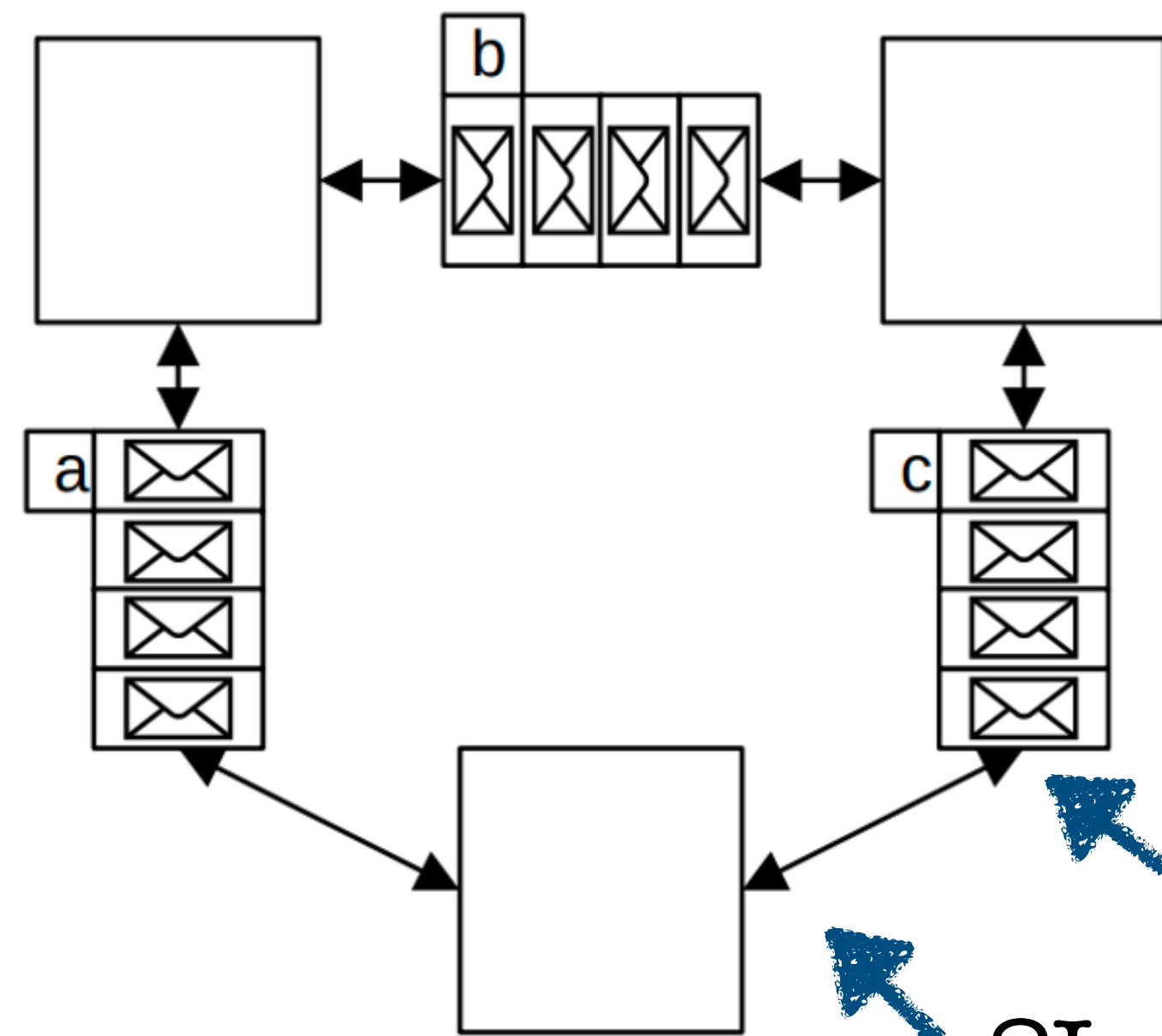
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STARDUST Meeting, University of Kent
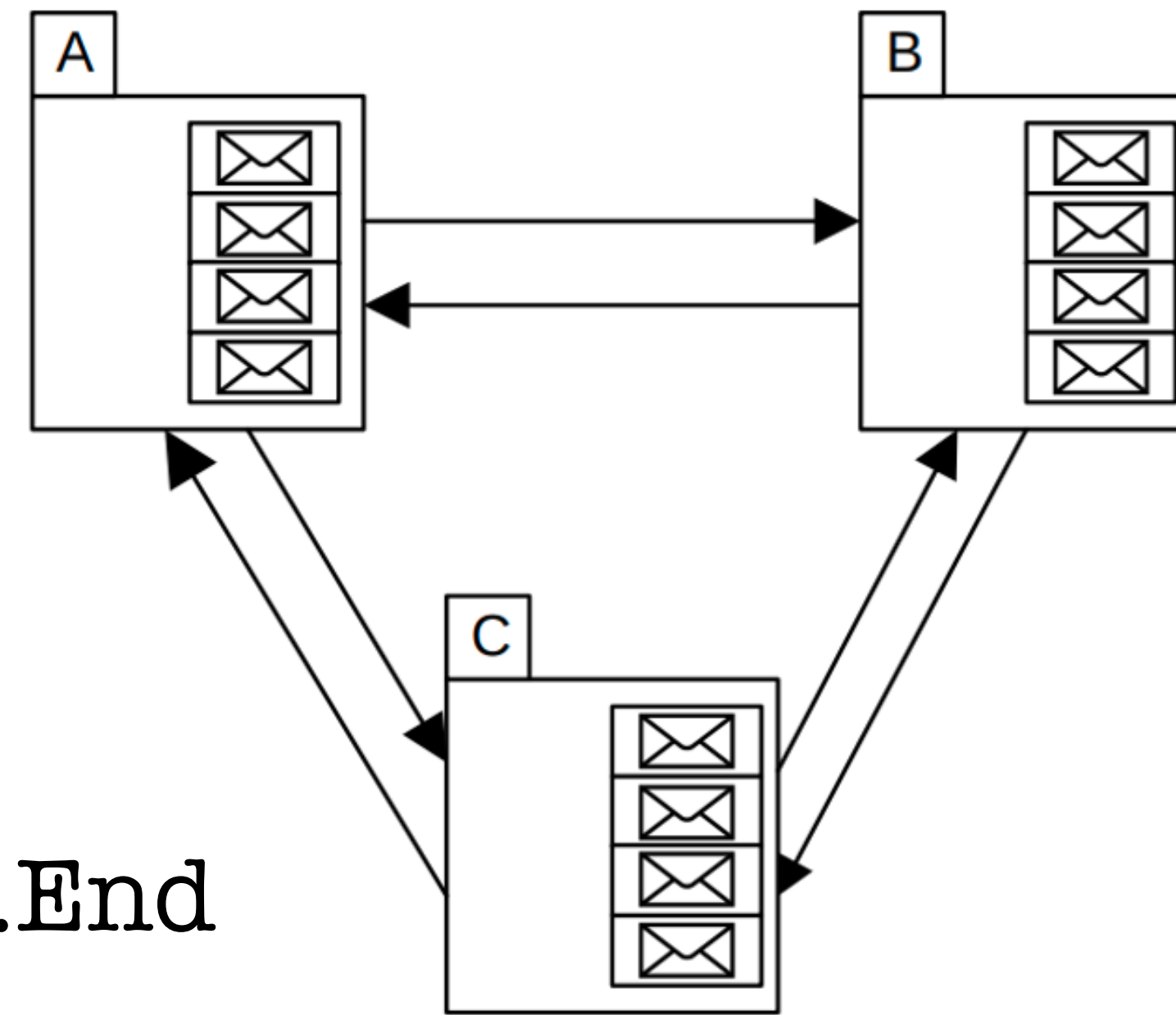
12th September 2022

STARDUST

University of Glasgow

VIA VERITAS VITA
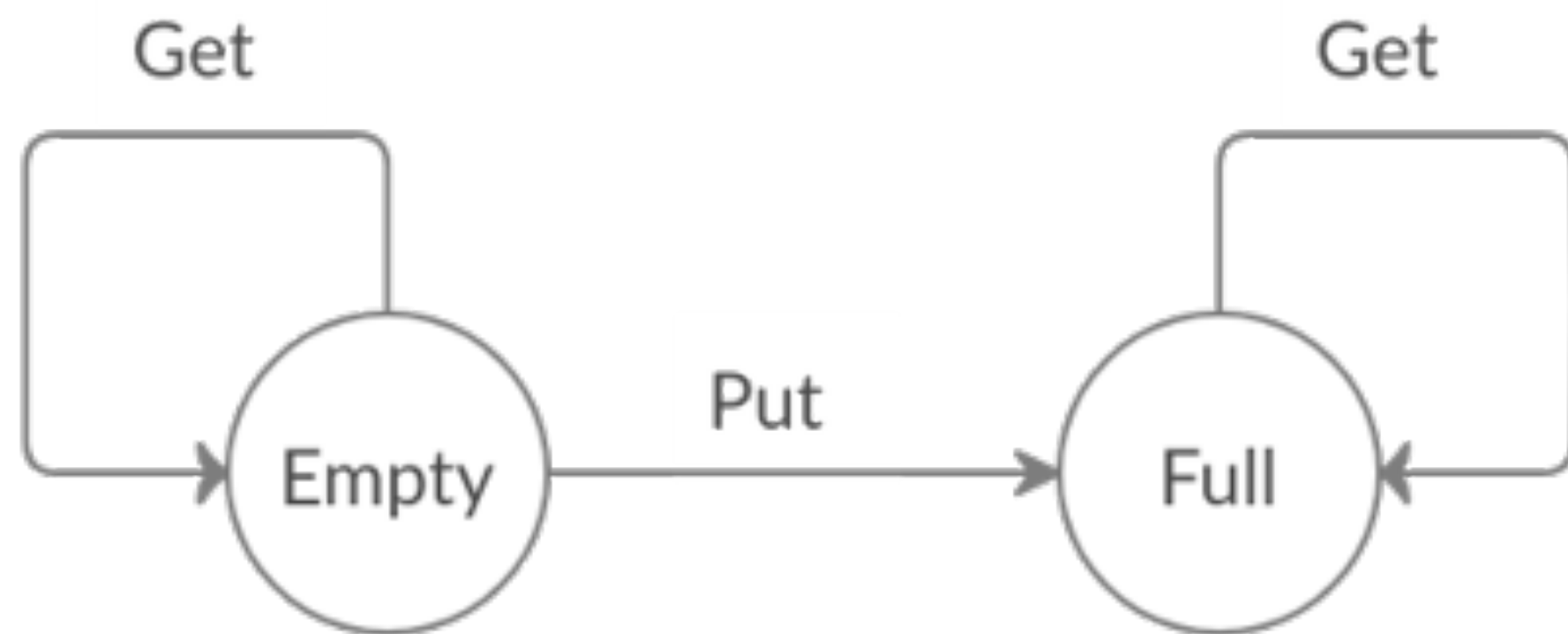
!Int.!Int.?Bool.End
?Int.?Int.!Bool.End

Channels

Actors

- Communication is **ordered** and **bidirectional**

- **Anonymous** processes, **multiple, named** channel endpoints

- Easy to type; difficult to distribute

- Communication is **unidirectional** and **possibly unordered** (selective receive)

- Named processes, associated with incoming **mailbox**

- Easy to distribute; difficult to type

Future: Placeholder variable

Can be written once, read many times

Multiple writes: error

```erlang
empty_future() ->
    receive
        { put, X } -> full_future(X)
    end.
```

```erlang
full_future(X) ->
    receive
        { get, Pid } ->
            Pid ! { reply, X },
            full_future(X);
        { put, _ } ->
            erlang:error("Multiple writes")
    end.
```

```erlang
main() ->
    Future = spawn(future, empty_future, []),
    Future ! { put, 5 },
    Future ! { get, self() },
    receive
        { reply, Result } ->
            io:fwrite("~w~n", [Result + 10])
    end.
```

Protocol violation
Two 'put' messages.
Manifests as a runtime error.

```erlang
empty_future() ->
  receive
    { put, X } -> full_future(X)
  end.

full_future(X) ->
  receive
    { get, Pid } ->
      Pid ! { reply, X },
      full_future(X);
    { put, _ } ->
      erlang:error("Multiple writes")
  end.

main() ->
  Future = spawn(future, empty_future, []),
  Future ! { put, 5 },
  Future ! { put, 10 },
  Future ! { get, self() },
  receive
    { reply, Result } ->
      io:fwrite("~w~n", [Result + 10])
  end.
```

Protocol violation
No 'put' message.
Future never resolved.

```erlang
empty_future() ->
  receive
    { put, X } -> full_future(X)
  end.

full_future(X) ->
  receive
    { get, Pid } ->
      Pid ! { reply, X },
      full_future(X);
    { put, _ } ->
      erlang:error("Multiple writes")
  end.

main() ->
  Future = spawn(future, empty_future, []),
  Future ! { get, self() },
  receive
    { reply, Result } ->
      io:fwrite("~w~n", [Result + 10])
  end.
```

Protocol violation
No 'reply' message.
Requests go unanswered.

```erlang
empty_future() ->
    receive
      { put, X } -> full_future(X)
    end.

full_future(X) ->
    receive
      { get, Pid } ->
        full_future(X);
      { put, _ } ->
        erlang:error("Multiple writes")
    end.

main() ->
    Future = spawn(future, empty_future, []),
    Future ! { put, 5 },
    Future ! { get, self() },
    receive
      { reply, Result } ->
        io:fwrite("~w~n", [Result + 10])
    end.
```

Unexpected Message
Message is never handled.

```erlang
empty_future() ->
    receive
        { put, X } -> full_future(X)
    end.

full_future(X) ->
    receive
        { get, Pid } ->
            Pid ! { reply, X },
            full_future(X);
        { put, _ } ->
            erlang:error("Multiple writes")
    end.

main() ->
    Future = spawn(future, empty_future, []),
    Future ! { put, 5 },
    Future ! { surprise, 10 },
    Future ! { get, self() },
    receive
        { reply, Result } ->
            io:fwrite("~w~n", [Result + 10])
    end.
```

Payload Mismatch
Client code expects an integer; gets a string.

```erlang
empty_future() ->
    receive
        { put, X } -> full_future(X)
    end.

full_future(X) ->
    receive
        { get, Pid } ->
            Pid ! { reply, X },
            full_future(X);
        { put, _ } ->
            erlang:error("Multiple writes")
    end.

main() ->
    Future = spawn(future, empty_future, []),
    Future ! { put, "hello" },
    Future ! { get, self() },
    receive
        { reply, Result } ->
            io:fwrite("~w~n", [Result + 10])
    end.
```

Self-deadlock

Attempting to read a reply message before sending a request.

```erlang
empty_future() ->
    receive
        { put, X } -> full_future(X)
    end.

full_future(X) ->
    receive
        { get, Pid } ->
            Pid ! { reply, X },
            full_future(X);
        { put, _ } ->
            erlang:error("Multiple writes")
    end.

main() ->
    Future = spawn(future, empty_future, []),
    Future ! { put, 5 },
    receive
        { reply, Result } ->
            io:fwrite("~w~n", [Result + 10])
    end,
    Future ! { get, self() }.
```

Mailbox Types for Unordered Interactions

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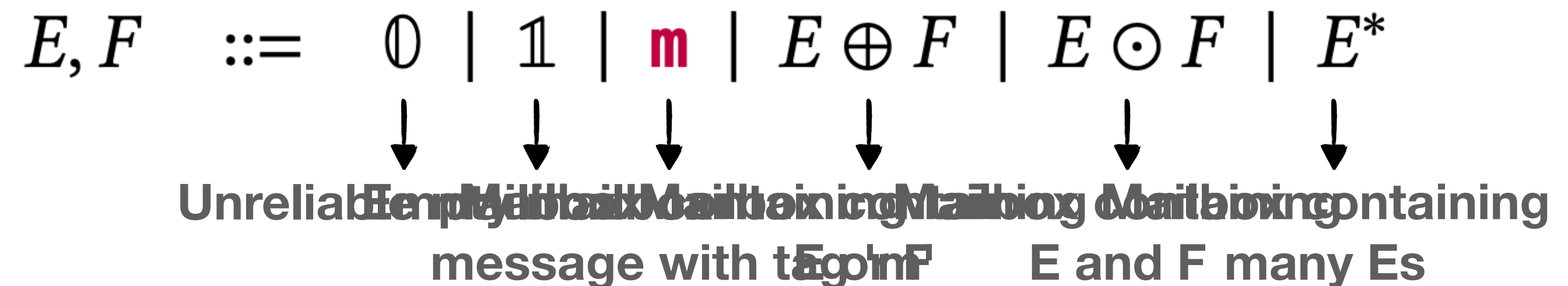https://orcid.org/0000-0001-9097-1297

—— Abstract ——
We propose a type system for reasoning on protocol conformance and deadlock freedom in networks of processes that communicate through unordered mailboxes. We model these networks in the mailbox calculus, a mild extension of the asynchronous π-calculus with first-class mailboxes and selective input. The calculus subsumes the actor model and allows us to analyze networks with dynamic topologies and varying number of processes possibly mixing different concurrency abstractions. Well-typed processes are deadlock free and never fail because of unexpected messages. For a non-trivial class of them, junk freedom is also guaranteed. We illustrate the expressiveness of the calculus and of the type system by encoding instances of non-uniform, concurrent objects, binary sessions extended with joins and forks, and some known actor benchmarks.

Mailbox Types: Type mailboxes with **commutative regular expressions**

$$E, F \quad ::= \quad \mathbb{0} \mid \mathbb{1} \mid \mathbf{m} \mid E \oplus F \mid E \odot F \mid E^*$$

Unreliable Empty Mailbox Mailbox containing Mailbox containing Mailbox containing
message with tag **m** E or F E and F many Es

A **mailbox type** is a **capability** associated with a **pattern**:

$$!E \qquad ?E$$

Key ideas:
- Each mailbox has **many** send references, but **precisely one** receive reference
- Sends and receives must balance out
- Subtyping: relies on **pattern inclusion**

$$\text{EmptyFuture} \triangleq ?(\textbf{Put}[\text{Int}] \odot \textbf{Get}[\text{ClientSend}]^*)$$

$$\text{FullFuture} \triangleq ?\textbf{Get}[\text{ClientSend}]^*$$

$$\text{ClientRecv} \triangleq ?\textbf{Reply}[\text{Int}]$$

$$\text{ClientSend} \triangleq !\textbf{Reply}[\text{Int}]$$

$$
\begin{aligned}
\text{emptyFuture}(\textit{self}) \quad &\triangleq \quad \textit{self}?\textbf{Put}(x).\text{fullFuture}(\textit{self}, x) \\
\text{fullFuture}(\textit{self}, x) \quad &\triangleq \quad \textbf{free}\ \textit{self}.\textbf{done} \\
&+ \quad \textit{self}?\textbf{Get}(\textit{sender}).(\textit{sender}!\textbf{Reply}[x] \ \| \ \text{fullFuture}(\textit{self}, x)) \\
&+ \quad \textit{self}?\textbf{Put}(x).\textbf{fail}\ \textit{self}
\end{aligned}
$$

$$
\begin{aligned}
(\textit{vfuture})(\text{emptyFuture}(\textit{future}) \ \| \ \textit{future}!\textbf{Put}[5] \ \| \\
(\textit{vself})(\textit{future}!\textbf{Get}[\textit{self}] \ \| \ (\textit{self}?\textbf{Reply}(x).\textbf{free}\ \textit{self}.\text{print}(\text{intToString}(x)))
\end{aligned}
$$

A process calculus shows a **snapshot of a concurrent system**

A programming language must be able to describe the **program a user writes**

```
def emptyFuture(self : EmptyFuture): 1 {
  guard self {
    receive Put [x] from self ↦ fullFuture(self, x)
  }
}

def fullFuture(self : FullFuture, value : Int): 1 {
  guard self {
    free ↦ ()
    receive Get [user] from self ↦
      user ! Reply [value] ;
      fullFuture(self, value)
  }
}

def client(): 1 {
  let future = new in
  spawn emptyFuture(future);
  let self = new in
  future ! Put [5] ;
  future ! Get [self] ;
  guard self {
    receive Reply [result] from self ↦
      free self;
      print(intToString(result))
  }
}
```

Demo

Language Integration

Challenge 1: Static / Dynamic Distinction

$$(vfuture)(\text{emptyFuture}(future) \parallel future\,!\,\textbf{Put}\,[\,5\,] \parallel$$
$$(vself)(future\,!\,\textbf{Get}\,[\,self\,] \parallel (self\,?\,\textbf{Reply}(x)\,.\,\textbf{free}\;self\,.\,\text{print}(\text{intToString}(x)))$$

Names

- Process calculus: know runtime names *a priori* as they are part of a process
- In a PL: only *dynamic*: generated by the semantics.
- Distinction incompatible with alias control / deadlock-freedom techniques used by the mailbox calculus

Sequential composition?

Variable rebinding?

Challenge 2: Name hygiene

$\textbf{def } \text{useAfterFree}(x : \textbf{?Message}[1]^*): 1 \{$

$\quad \textbf{guard } x \{$

$\quad\quad \textbf{receive Message}[y] \textbf{ from } z \mapsto$

$\quad\quad\quad x\,!\,\textbf{Message}[()];$

$\quad\quad\quad \text{useAfterFree}(z)$

$\quad\quad \textbf{free} \mapsto$

$\quad\quad\quad \boxed{x\,!\,\textbf{Message}[()]}$

$\quad \}$

$\}$

A guard 'uses up' a variable; x must not be in scope afterwards

Easy to do in a linear system; more difficult in a multi-writer system where variables can be used more than once

Challenge 2: Name hygiene

```
def useAfterFree(x : ?Message[1]*): 1 {
    let a = x in
    guard a {
        receive Message[y] from z ↦
            x!Message[()];
            useAfterFree(z)
        free ↦
            x!Message[()]
    }
}
```

```
def useAfterFree(x : ?Message[1]*): 1 {
    let _ =
        guard x {
            receive Message[y] from z ↦
                x!Message[()];
                useAfterFree(z)
            free ↦
                x!Message[()]
        }
    in x!Message[()]
}
```

...and must be robust to renaming / aliasing, and evaluation contexts!

Need: Only one variable name in scope for each runtime name

Challenge 3: Aliasing via Communication

$$a \leftarrow \mathsf{m}[b] \quad \| \quad \begin{array}{l} \textbf{guard } a \ \{ \\ \quad \textbf{receive } \mathsf{m}[x] \ \textbf{from } y \mapsto \\ \qquad b\,!\,\mathsf{n}[x]; \\ \qquad \textbf{free } y \\ \} \end{array} \quad \longrightarrow \quad \begin{array}{l} b\,!\,\mathsf{n}[b]; \\ \textbf{free } a \end{array}$$

Cannot allow communication to introduce
unsafe aliases!

Quasi-Linear Types

- Quasi-linear typing: each reference can be used **once per process** as a full ("returnable") reference, but many times as a partial ("usable") reference

- Returnable references can be let-bound; returned as part of an expression; and guarded upon

- Returnable reference must be the **last occurrence** of the name in the thread

- Usable references can only be used as the target of a send

Quasi-Linear Types: Example


```
def client(): 1 {
    let future = new in
    spawn emptyFuture(future);
    let self = new in                    !Reply°
    future ! Put [5];
    future ! Get [self];                 ?Reply•
    guard self {
        receive Reply [result] from self ↦
            free self;                   ?1•
        print(intToString(result))
    }
}
```

Typable: Returnable use **always last in scope** (note that 'self' is consumed by 'guard' and rebound)

Quasi-Linear Types: Example

$$\textbf{def } \text{useAfterFree}(x : (\textbf{?Message}[1]^*)^\bullet) : 1 \{$$

$$\textbf{let } a = \boxed{x} \textbf{ in}$$

$$\textbf{guard } a \{ \qquad \qquad ?(\text{Message}^*)^\bullet$$

$$\textbf{receive Message}[y] \textbf{ from } z \mapsto$$

$$!\text{Message}^\circ \longleftarrow \boxed{x}!\textbf{Message}[()] ;$$

$$\text{useAfterFree}(z)$$

$$\textbf{free} \mapsto$$

$$!\text{Message}^\circ \longleftarrow \boxed{x}!\textbf{Message}[()]$$

$$\}$$

$$\}$$

Untypable: variable 'x' appears *after* returnable occurrence

Mailbox types	J, K	$::=$	$!E \mid \ ?E$
Mailbox patterns	E, F	$::=$	$\mathbb{0} \mid \mathbb{1} \mid \mathsf{m} \mid E \oplus F \mid E \odot F \mid E^*$
Base types	C	$::=$	$1 \mid \mathsf{Int} \mid \mathsf{String} \mid \cdots$
Types	T, U	$::=$	$C \mid J$
Usage annotations	η	$::=$	$\circ \mid \bullet$
Usage-annotated types	A, B	$::=$	$C \mid J^\eta$

Variables	x, y, z		
Definition names	f		
Definitions	D	$::=$	$\mathbf{def}\ f(\overrightarrow{x : A}): B\ \{M\}$
Values	V, W	$::=$	$x \mid c$
Terms	L, M, N	$::=$	$V \mid \mathbf{let}\ x : T = M\ \mathbf{in}\ N \mid f(\overrightarrow{V})$
		$\mid$	$\mathbf{spawn}\ M \mid \mathbf{new} \mid V\,!\,\mathsf{m}[\overrightarrow{W}] \mid \mathbf{guard}\ V\ \{\overrightarrow{G}\}$
Guards	G	$::=$	$\mathbf{fail} \mid \mathbf{free} \mapsto M \mid \mathbf{receive}\ \mathsf{m}[\overrightarrow{x}]\ \mathbf{from}\ y \mapsto M$

Selected Typing Rules (Send)

Message with tag **m** has payload types $\overrightarrow{T}$ Target must have usable mailbox type ! **m**

Payload types must match (usable)

$$\frac{\mathcal{S}(\textbf{m}) = \overrightarrow{T} \qquad \Gamma \vdash V : \, !\,\textbf{m}^\circ \qquad (\Gamma'_i \vdash W_i : \lceil T_i \rceil)_{i \in 1..n}}{\Gamma + \Gamma'_1 + \ldots + \Gamma'_n \vdash V\,!\,\textbf{m}[\overrightarrow{W}] : 1}$$

Send term has **unit type**, no shared linear variables between target and each payload

Selected Typing Rules (Let)

Ensure subject of **let** has **returnable** type

$$\frac{\Gamma_1 \vdash M : \lfloor T \rfloor \qquad \Gamma_2, x : \lfloor T \rfloor \vdash N : B}{\Gamma_1 \triangleright \Gamma_2 \vdash \textbf{let } x : T = M \textbf{ in } N : B}$$

Sequencing of environments: ensures mailbox types combine correctly, ensure quasilinear well-formedness properties

Note: Type annotation **optional**

Type Combination

$$!E \boxplus !F = !(E \odot F)$$

$$!E \boxplus ?(E \odot F) = ?F \qquad ?(E \odot F) \boxplus !E = ?F$$

Combining two send mailbox types: send **both**

Combining a send and receive: types **balance out**
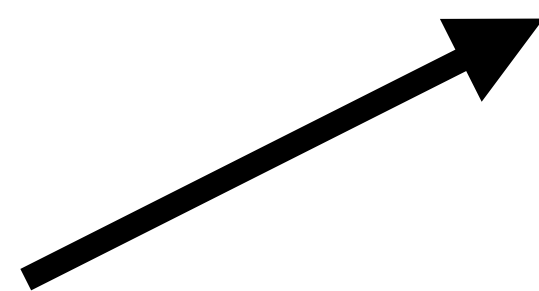
$$\circ \triangleright \circ = \circ$$

$$\circ \triangleright \bullet = \bullet$$

Can combine two usable types, or a usable and a returnable type
Note: Not reflexive, nor symmetric
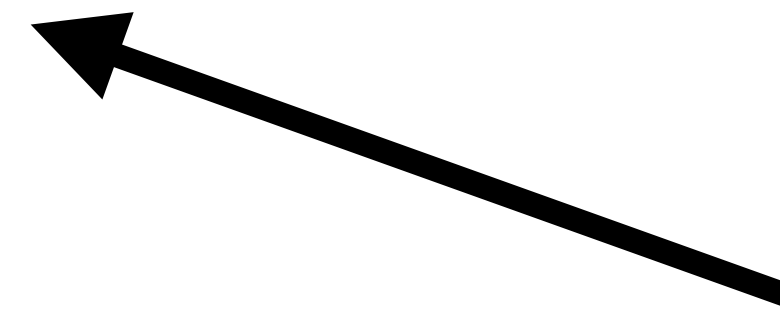
Selected Typing Rules (New and Spawn)

$$\frac{}{\cdot \vdash \mathbf{new} : \boxed{?\mathbb{1}^{\bullet}}}$$

New mailbox: Returnable, empty
receive capability.
Ensures all sends balanced by receives

$$\frac{\Gamma \vdash M : \mathbf{1}}{\boxed{\lceil \Gamma \rceil} \vdash \mathbf{spawn}\ M : \mathbf{1}}$$

Spawn: environment treated as **usable**
(quasi-linearity is **thread-local**)

Metatheory

Theorem (Preservation):
If Γ is reliable, $\Gamma \vdash \mathscr{C}$, and $\mathscr{C} \longrightarrow \mathscr{D}$, then $\Gamma \vdash \mathscr{D}$.

Corollary (Mailbox Conformance):
If Γ is reliable and $\Gamma \vdash \mathscr{C}$, then $\mathscr{C} \not\longmapsto^* \mathscr{G}[\mathbf{fail}\ V]$.

Nontrivial: requires extensive reasoning about contexts and quasi-linearity

Algorithmic Typing & Implementation

How do we write a typechecker?

The declarative system cannot be implemented as-is:

- Nondeterministic environment & type splitting

- Environment subtyping

- Pattern inclusion

Key idea: **Produce** a type environment and a set of pattern inclusion constraints

Bidirectional Typing

$$\Gamma \vdash M \Rightarrow A$$

*"Under type environment Γ, we can **synthesise** type A for term M"*

$$\Gamma \vdash M \Leftarrow A$$

*"Under type environment Γ, we can **check** that term M has type A"*

Backwards Bidirectional Typing

$$P \Rightarrow \tau \blacktriangleright \Theta; \Phi$$

"Synthesise type τ for term P, ***producing*** *type environment Θ and pattern inclusion constraints Φ"*

$$P \Leftarrow \tau \blacktriangleright \Theta; \Phi$$

"Check that term P has type type τ, ***producing*** *type environment Θ and pattern inclusion constraints Φ"*

Key idea: Stay in checking mode as much as possible to preserve type information and propagate to variables
(originally introduced by Zeilberger, 2015)

$$\text{TC-Var}$$
$$\overline{}$$
$$x \Longleftarrow \tau \blacktriangleright x : \tau; \emptyset$$

Variable rule: a **checking** case, constructing a singleton environment

Lookup payload types for message tag **m**

Check payloads have correct types

Check target can send message with tag **m**

TS-Send

$$\frac{\mathcal{S}(\mathsf{m}) = \overrightarrow{\pi} \qquad V \Leftarrow \, ! \, \mathsf{m}^{\circ} \, \blacktriangleright \, \Theta'; \Phi \qquad (W_i \Leftarrow \lceil \pi_i \rceil \, \blacktriangleright \, \Theta_i'; \Phi_i')_{i \in 1..n} \qquad \Theta' + \Theta_1' + \cdots + \Theta_n' \, \blacktriangleright \, \Theta; \Phi''}{V \, ! \, \mathsf{m} [\overrightarrow{W}] \Rightarrow \mathbf{1} \, \blacktriangleright \, \Theta; \Phi \cup \Phi_1' \cup \cdots \cup \Phi_n' \cup \Phi''}$$

Synthesise unit type for the send term, producing combined environment Theta and the union of all constraint sets

Calculate disjoint combination of produced typing environments

Check that the body has correct produced type
Check that the body types environment, see that x has type τ

Ensure that τ is returnable

$$Q \Longleftarrow \sigma \blacktriangleright \Theta_1, x : \tau; \Phi_1 \qquad returnable(\tau)$$

$$P \Longleftarrow \tau \blacktriangleright \Theta_2; \Phi_2 \qquad \Theta_2 \mathbin{\overset{\circ}{\circ}} \Theta_1 \blacktriangleright \Theta; \Phi_3$$

$$\overline{}$$

$$\textbf{\textit{let }} x = P \textbf{\textit{ in }} Q \Longleftarrow \sigma \blacktriangleright \Theta; \Phi_1 \cup \Phi_2 \cup \Phi_3$$

Check that P has type τ

Calculate algorithmic sequencing of environments

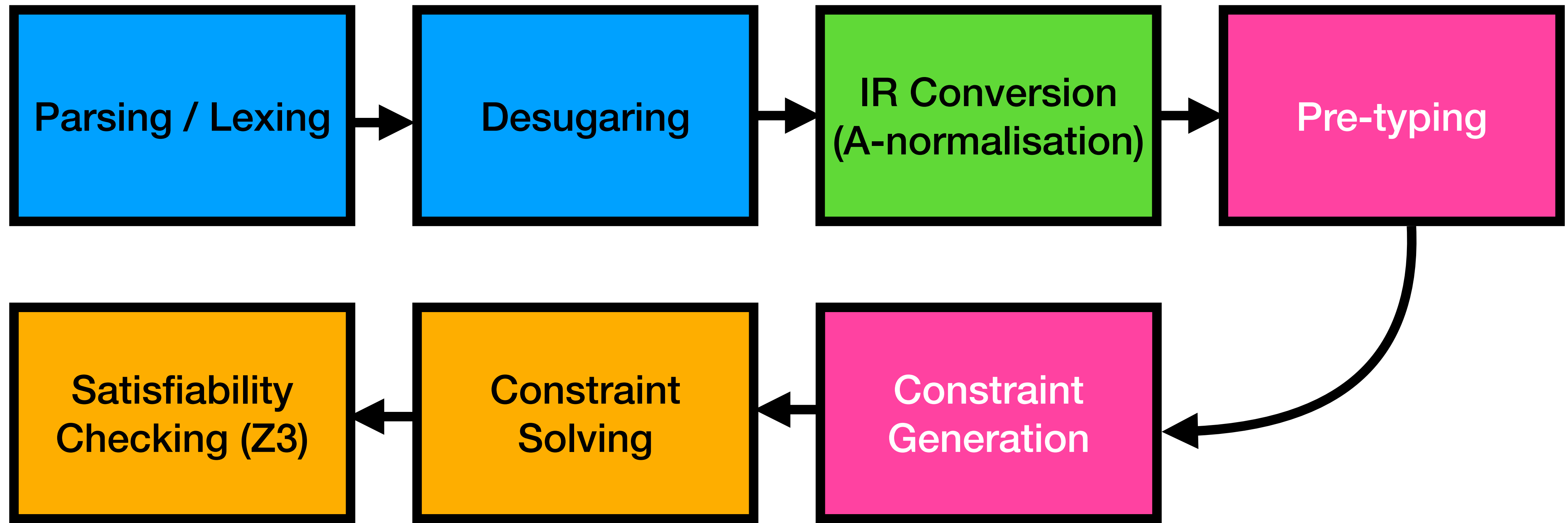**Note**: Revert to synthesis if x is not used in Q

Metatheory

Theorem (Algorithmic Soundness)

- If $P \Leftarrow \tau \blacktriangleright \Theta; \Phi$ and Ξ is a usable solution for Φ, then $\Xi(\Theta) \vdash \Xi(P) : \Xi(\tau)$

- If $P \Rightarrow \tau \blacktriangleright \Theta; \Phi$ and Ξ is a usable solution for Φ, then $\Xi(\Theta) \vdash \Xi(P) : \Xi(\tau)$

Conjecture (Algorithmic Completeness)

If $\Gamma \vdash M : A \rightsquigarrow P$, then there exist some Θ, Φ and a usable solution Ξ of Φ such that $P \Leftarrow A \blacktriangleright \Theta; \Phi$ where $\Gamma \leq \Xi(\Theta)$.

Implementation

Pattern inclusion: closed form solution thanks to Hopkins & Kozen (1999)
Check **consistency** by translating into Presburger formulae & offloading to Z3

Wrapping up

Mailbox Types: Type the **mailbox**, not the **process**

First integration of mailbox types into a programming language:

- Vital use of quasi-linear types to handle many-writer, single-reader pattern

Sound and complete algorithmic type system based on **backwards bidirectional typing**

Future work

- Compilation (Ongoing, with Franek Sowul)

- Constraint-based co-contextual typing algorithm

- More sophisticated alias analysis

- Tool integration (Erlang, Elixir...)

- Other many-writer paradigms (Publish-subscribe? Typestate?)